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A Trajectory Optimization Formulation for Assistive Robotic Devices

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Abstract— **We present a trajectory optimization formulation for the design of a sit-to-stand assistive device for humans with motion impairments. We develop a constrained Lagrangian to derive the equations of motion for the trajectory optimization formulation. Such a Lagrangian enables us to impose constraints on joint reaction wrenches in the human, to simulate the motion impairment of human joints due to injury and infirmity. Using trajectory optimization, we compute the optimal sit-to-stand motions of a human as well as the actuation wrenches of the sit-to-stand device. The trajectory of the device as it follows and supports the human, and its actuation wrenches provide necessary inputs for the design of the assistive device. We propose an assistive device capable of following the natural trajectory of the human during sit-to-stand. We show by numerical simulation that the human requires less effort with the device that can follow its trajectory than with an assistive device restricted to only vertical motion to lift up a human. Our formulation provides a systematic approach for the design of such sit-to-stand assistive devices.**

I. INTRODUCTION

Trajectory optimization is widely used for robot motion planning and control [32], [7], [3], [22], [29] and analysis of human motions [24], [37]. Our goal is to develop trajectory optimization formulations to model motion impaired humans and use the results of trajectory optimization, for example, velocities, accelerations, and actuation wrenches, to develop better designs of assistive devices. Sit-to-stand is an activity of daily living (ADL) that requires the human to expend much effort compared to other ADLs, for example, walking. However, whereas assistive devices for walking (e.g., walking sticks, walkers) are widely available, there are few commercially available sit-to-stand assistive devices that actually lower the effort of humans during sit-to-stand. This motivates our work on trajectory optimization of sit-to-stand of humans and the design of sit-to-stand assistive devices.

Our trajectory optimization formulation is based on a constrained Lagrangian from which we derive the state equations (equations of motion). We model the human as a system of five rigid links connected by revolute joints as shown in Figure 1(a). Using kinematic constraints we obtain the constraint wrenches (for example, reaction forces at a revolute joint), and in the trajectory optimization formulation we incorporate bounds on the constraint wrenches. Such constraints correspond to injured or impaired joints in the human. These constraints should ideally be satisfied along the trajectory; our current solution guarantees satisfaction of these constraints only at discrete points along the trajectory. Our focus is on trajectory optimization with constraints on these internal joint-reaction wrenches, rather than trajectory optimization in the presence of discontinuous external contact forces [29], [10].

Following a brief literature survey, we present our trajectory optimization formulation with the equations of motion. Using the trajectory optimization formulation, we model the sit-to-stand motion of a human including constraints on internal joint reaction wrenches. From the trajectory optimization results, we derive the actuation wrenches and the motion parameters (configuration, velocity, and acceleration) of our proposed sit-to-stand device to follow the human's motion and support the human to minimize its effort. We also compare the effort for sit-to-stand using our device and a model of a commercially available sit-to-stand device.

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II. RELATED WORK

Our trajectory optimization problem is a two-point boundary value problem with a fixed time duration, and the problem is to determine the trajectory that minimizes an objective function (e.g., work done) subject to a set of constraints (e.g., equations of motion). For a concise overview of trajectory optimization problems in robotics, see [7]. The two main methods used to solve trajectory optimization problems are indirect methods using optimality conditions [5], [4], and direct methods using nonlinear programming [14], [22], [29]. We use the direct approach as it is difficult to incorporate path inequality constraints in the indirect methods and their region of convergence is relatively small [22]. Our approach is similar to direct collocation methods [15], [35]. The recent discrete mechanics and optimal control (DMOC) approach [23] provides a finite dimensional nonlinear optimization problem solvable by sequential quadratic programming.

Robotic Assistive Aids: Assistive robotic aids for sit-to-stand and walking motions have been developed, including commercial lift walkers [33], [9] and research prototypes [19], [28], [11], [8], [20], [6]. Kamnik and Bajd [19] developed a robot assistive device for training standing-up in impaired people by supporting the subject under the buttocks. Peshkin et al. [28] developed KineAssist, an assistive robotic device for gait and balance training that provides partial body weight support. Fattah et al. [11] designed a passive gravity-balancing assist device for sit-to-stand motions. Chuy et al. [8] used a robotic walker to provide assistance during sit-to-stand by reducing the torque at the knee. Kim et al. [20] presented a kinematic analysis of a smart mobile walker for walking and sit-to-stand assistance. Bulea and

Triolo [6] developed a vertical lift walker for assistance with sit-to-stand and transition to walking, but did not consider dynamics of the human. The device of Purwar et al. [30] is a six-bar lifting apparatus that lifts a person along a natural path and maintains a constant orientation of the link of the device that lifts the user.

Dynamics in rehabilitation robotics: While much attention has focused on the kinematics of assistive devices, generating trajectories for the assistive robot and the human that are consistent with dynamics is also important. Wang, Bobrow, and Reinkensmeyer [36] developed a dynamics based optimal control formulation to generate robot motions to enable leg swinging motion of a paralyzed person. Kuzelicki et al. [21] developed an optimization formulation for the dynamics of unassisted sit-to-stand and compared it to experimental data. They conclude that the body dynamics and kinematics, without modeling muscle behavior, yield sufficiently accurate trajectories to compute trajectories for assistive robots. Yamasaki, Kambara, and Koike [39] found that experimental sit-to-stand trajectories can be predicted by a dynamic optimization model.

Dynamics and Optimal Motion: The dynamics formulations used in trajectory optimization methods for rigid multibody mechanisms can be divided into two broad groups, the Newton-Euler based methods, and the Lagrangian based methods [13], [38]. Uicker [34] first formulated the Lagrangian dynamics equations for a mechanism with n links, which required $O(n^4)$ computation. Hollerbach [16] developed a recursive Lagrangian formulation that requires only $O(n)$ computation. The Newton-Euler based algorithms for recursively calculating joint actuator forces and torques were developed in [1], [25], [26] for an open chain manipulator; these widely used algorithms require only $O(n)$ computation. Later, Featherstone [12] developed a spatial formulation and Park et al. [27] developed a Lie group formulation of the recursive Newton-Euler algorithm.

Nonlinear programming techniques are widely used in the optimal motion of humanoid robots and human models [32], [24], [22]. Lo and Metaxas [24] used Featherstone's recursive Newton-Euler algorithms to compute human motions for different tasks. Lee et al. [22] developed Park et al.'s [27] formulation to find optimal effort motions for robot and human models. Xiang et al. [37] used Hollerbach's recursive Lagrangian formulation to predict human gait under different loading conditions. However, all these models assume perfect physical condition of the human whereas we are interested in finding the optimal motion of a human with a limb injury, modeled as a force constraint at the actuating joints of the limb.

III. Problem and Approach

We are interested in computing an optimal trajectory with actuation wrenches for a multibody system subject to dynamics constraints, and additionally, joint reaction force constraints at specified joints.

A. The Lagrangian with Constraints

Consider the Lagrangian with constraints:

$$\begin{aligned}\mathcal{L} &= K - V + W + \sum_k \lambda_k \phi_k \\ &= \sum_{j=1}^{n} \Big[\frac{m_j(v_{xj}^2 + v_{yj}^2)}{2} + \frac{I_j \dot{\theta}_j^2}{2} - m_j g y_j + \tau_j(\theta_j - \theta_{j-1}) \Big] \\ &+ \sum_k \lambda_k \phi_k \end{aligned} \tag{1}$$

where K is the kinetic energy, V is the potential energy, W is the work done by the actuation wrenches, and λ_k is the Lagrange multiplier for the internal kinematic constraint ϕ_k. The number of bodies in the chain is n, v_{xj} and v_{yj} are the X and Y components of the velocity of the center of mass (COM) of body j with coordinates (x_j, y_j). m_j and I_j are the mass and mass moment of inertia (computed about the COM of body j in the world reference frame) of body j, τ_j is the torque acting at joint j, r_j is the distance of the COM of body j from joint j, and l_j is the length of body j. We use the notation $c_j = \cos(\theta_j)$, $s_j = \sin(\theta_j)$, and $c_{j-i} = \cos(\theta_j - \theta_i)$. So $y_j = r_j s_j + \sum_{k=1}^{j-1} l_k s_k$.

Let i be the link where a kinematic constraint is imposed (Figure 1(a)). Here the Lagrange multipliers λ_k are the joint reaction forces F_{x_i} and F_{y_i} acting on link i at the revolute joint i. Then the Lagrangian in Equation (1) will have the following form:

$$\begin{aligned}\mathcal{L} &= \sum_{j=1}^{n} \Big[\frac{m_j(v_{xj}^2 + v_{yj}^2)}{2} + \frac{I_j \dot{\theta}_j^2}{2} - m_j g y_j + \tau_j(\theta_j - \theta_{j-1}) \Big] \\ &+ F_{x_i} \left[x_i - r_i c_i - \sum_{k=1}^{i-1} l_k c_k \right] + F_{y_i} \left[y_i - r_i s_i - \sum_{k=1}^{i-1} l_k s_k \right] \end{aligned} \tag{2}$$

When we extremize the Lagrangian, we get the equations of motion of the system:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0 \tag{3}$$

where q_i is the ith generalized coordinate. For an open chain, the actuation torque τ_i at joint i is:

$$\begin{aligned}\tau_i &= I_i \ddot{\theta}_i + m_i \left[\sum_{k=1}^{i-1} r_i l_k (s_{i-k} \dot{\theta}_k^2 + c_{i-k} \ddot{\theta}_k) + r_i^2 \ddot{\theta}_i \right] \\ &+ \sum_{j=i+1}^{n} m_j \left[\sum_{k=1}^{j-1} l_i l_k (s_{i-k} \dot{\theta}_k^2 + c_{i-k} \ddot{\theta}_k) \right] \\ &+ \sum_{j=i+1}^{n} m_j \left[r_j l_i (c_{j-i} \ddot{\theta}_j - s_{j-i} \dot{\theta}_j^2) \right] \\ &+ \left(m_i r_i + l_i \sum_{j=i+1}^{n} m_j \right) g c_i + \tau_{i+1} \end{aligned} \tag{4}$$

When we extremize the Lagrangian in Equation (2) we get the following equations of motion:

$$
\begin{aligned}
F_{x_i} &= m_i \ddot{x}_i \\
&= -m_i \left[\sum_{j=1}^{i-1} l_j \{ c_j \dot{\theta}_j^2 + s_j \ddot{\theta}_j \} + r_i \{ c_i \dot{\theta}_i^2 + s_i \ddot{\theta}_i \} \right]
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
F_{y_i} &= m_i \ddot{y}_i + m_i g \\
&= m_i \left[\sum_{j=1}^{i-1} l_j \{ -s_j \dot{\theta}_j^2 + c_j \ddot{\theta}_j \} + r_i \{ -s_i \dot{\theta}_i^2 + c_i \ddot{\theta}_i \} + g \right]
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
\tau_i = I_i \ddot{\theta}_i &+ \left(\sum_{j=i+1}^{n} m_j \right) g l_i c_i + F_{y_i} r_i c_i - F_{x_i} r_i s_i + \tau_{i+1} \\
&+ \sum_{j=i+1}^{n} m_j \left[\sum_{k=1}^{j-1} l_i l_k (s_{i-k} \dot{\theta}_k^2 + c_{i-k} \ddot{\theta}_k) \right] \\
&+ \sum_{j=i+1}^{n} m_j \left[r_j l_i (c_{j-i} \ddot{\theta}_j - s_{j-i} \dot{\theta}_j^2) \right]
\end{aligned} \tag{7}
$$

For joints $j < i$:

$$
\begin{aligned}
\tau_j = & I_j \ddot{\theta}_j + m_j \left[\sum_{k=1}^{j-1} r_j l_k (s_{j-k} \dot{\theta}_k^2 + c_{j-k} \ddot{\theta}_k) + r_j^2 \ddot{\theta}_j \right] \\
&+ \sum_{\substack{p=j+1 \\ p \neq i}}^{n} m_p \left[\sum_{k=1}^{p-1} l_j l_k (s_{j-k} \dot{\theta}_k^2 + c_{j-k} \ddot{\theta}_k) \right] \\
&+ \sum_{\substack{p=j+1 \\ p \neq i}}^{n} m_p \left[r_p l_j (c_{p-j} \ddot{\theta}_p - s_{p-j} \dot{\theta}_p^2) \right] \\
&+ \left(m_j r_j + l_j \sum_{p=j+1}^{n} m_p \right) g c_j \\
&+ F_{y_i} l_j c_j - F_{x_i} l_j s_j + \tau_{j+1}
\end{aligned} \tag{8}
$$

For joints $i < j \leq n$, τ_j is given by Equation (4).

For closed chains, the kinematic constraints are similar to those used in the constrained Lagrangian formulation (Equation (2)). Thus, if the coordinates of the joint $n+1$ closing the chain are (x_{n+1}, y_{n+1}), then the terms corresponding to the closed chain loop-closure constraint that are introduced in the Lagrangian are:

$$
F_{x_{n+1}} \left(x_{n+1} - \sum_{i=1}^{n} l_i c_i \right) + F_{y_{n+1}} \left(y_{n+1} - \sum_{i=1}^{n} l_i s_i \right) \tag{9}
$$

By extremizing the Lagrangian of Equation (2), the actuation torque at joint j in a closed chain is:

$$
\tau_j = \tau_j^{(4)} + F_{y_{n+1}} l_j c_j - F_{x_{n+1}} l_j s_j \tag{10}
$$

where $\tau_j^{(4)}$ denotes the expression for torque τ_j in Equation (4).

B. The Trajectory Optimization Formulation

The general form of the fixed time trajectory optimization problem for a system of rigid bodies [5] is

$$
\begin{aligned}
&\text{Minimize:} \int_{T_0}^{T_f} f(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\tau}) dt \\
&\text{Subject to:} \, \boldsymbol{M}(\boldsymbol{q}) \ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{\tau} \\
&\{ \boldsymbol{q}_{lb}, \dot{\boldsymbol{q}}_{lb}, \ddot{\boldsymbol{q}}_{lb} \} \leq \{ \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}} \} \leq \{ \boldsymbol{q}_{ub}, \dot{\boldsymbol{q}}_{ub}, \ddot{\boldsymbol{q}}_{ub} \} \\
&\boldsymbol{\tau}_{lb} \leq \boldsymbol{\tau} \leq \boldsymbol{\tau}_{ub} \\
&\{ \boldsymbol{q}(T_0), \dot{\boldsymbol{q}}(T_0), \ddot{\boldsymbol{q}}(T_0) \} = \{ \boldsymbol{q}_0, \dot{\boldsymbol{q}}_0, \ddot{\boldsymbol{q}}_0 \} \\
&\{ \boldsymbol{q}(T_f), \dot{\boldsymbol{q}}(T_f), \ddot{\boldsymbol{q}}(T_f) \} = \{ \boldsymbol{q}_{T_f}, \dot{\boldsymbol{q}}_{T_f}, \ddot{\boldsymbol{q}}_{T_f} \}
\end{aligned} \tag{11}
$$

where $\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}$ are the coordinates, velocities, and accelerations respectively, the objective function $f(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\tau})$, has to be minimized subject to the equations of motion, $\boldsymbol{\tau}$ are the actuator wrenches, and T_0 and T_f are the initial and final times respectively. $\boldsymbol{M}(\boldsymbol{q})$ is the inertia matrix, $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is comprised of the centripetal and Coriolis terms, and $\boldsymbol{G}(\boldsymbol{q})$ consists of conservative forces like gravity. There can be bounds on both configuration variables and actuation wrenches.

The optimal control problem in Equation (11) with continuous function variables is converted into a numerical optimization problem with a discrete set of variables by using cubic B-splines to represent C^2 continuous trajectories for the system [2]. We use an open uniform cubic B-spline with knot multiplicity of 4 at the two ends of the spline to ensure the spline passes through the first and last control points, and that the derivatives and double derivatives at the end points can be specified. The variables in the optimization are the control points of the cubic B-splines for the joint angles ($\boldsymbol{\theta}^k$) and the joint reaction wrenches ($F^k_{\{x,y\}_i}$); for a closed chain we additionally have reaction forces ($F^k_{\{x,y\}_{n+1}}$) for the joint $n+1$ that closes the loop.

The resulting nonlinear optimization problem with objective function Ψ and nonlinear constraints $\boldsymbol{\Phi}$ is:

$$
\begin{aligned}
&\text{Minimize:} \int_{T_0}^{T_f} f(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \ddot{\boldsymbol{\theta}}, \boldsymbol{\tau}) dt = \Psi(\{ \theta_j^k, F^k_{\{x,y\}_i} \}) \\
&\text{Subject to:} \\
&\boldsymbol{\Phi}(\{ \theta_j^k, F^k_{\{x,y\}_i} \}) = \boldsymbol{M}(\boldsymbol{q}) \ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) - \boldsymbol{\tau} = \boldsymbol{0} \\
&\boldsymbol{\theta}_{lb} \leq \boldsymbol{\theta}^k \leq \boldsymbol{\theta}_{ub} \\
&\boldsymbol{F}_{\{x,y\}_{lb}} \leq \boldsymbol{F}^k_{\{x,y\}} \leq \boldsymbol{F}_{\{x,y\}_{ub}} \\
&\{ \theta_j, \dot{\theta}_j, \ddot{\theta}_j \}|_{t=0} = \{ \theta_{j_0}, \dot{\theta}_{j_0}, \ddot{\theta}_{j_0} \} \\
&\{ \theta_j, \dot{\theta}_j, \ddot{\theta}_j \}|_{t=T_f} = \{ \theta_{j_f}, \dot{\theta}_{j_f}, \ddot{\theta}_{j_f} \}
\end{aligned} \tag{12}
$$

If there are N_c control points and N_b bodies, then there are $N_b N_c$ variables for a trajectory optimization problem; if the joint reaction forces at N_j joints are to be determined (Equation (2)), then there are 2 additional variables for every joint, and hence $2 N_j N_c$ more variables in the trajectory optimization problem (Equation (12)). Also there are $2 N_j$ nonlinear constraint equations given by Equations (5) and (6).

If the loop is closed then there are 2 additional variables for every closed-loop, and hence $2N_c$ more variables than for the open-loop trajectory optimization problem. For every closed-loop, there are 2 nonlinear loop-closure equations. The initial and final configurations, velocities, and accelerations form a set of $6N_b$ linear constraint equations. Joint angle limits are formulated as linear constraints. Note that since we use additional variables $F^k_{\{x,y\}_i}$ to model the internal joint reaction forces with cubic B-splines, the convex hull property of B-splines ensures that satisfaction of force limits at the force control points guarantees that force constraints are satisfied throughout the trajectory. We use the SNOPT solver [14], based on sequential quadratic programming, from MATLAB to solve the nonlinear optimization problem.

IV. Trajectory Optimization for Sit-to-Stand Assistive Device

We now show how the results of our trajectory optimization can help in the conceptual design of an assistive device to help patients and the elderly during sit-to-stand and stand-to-sit motions. Standing up stably from a bed or chair is one of the most challenging ambulatory tasks elderly people face. Standing up requires a high torque at the hip and knee, and this depends directly on muscle strength [18], [24]. It is also an activity of daily living that can lead to falls. Assistive devices that help elderly people to stand up from sitting positions increase their independence and reduce the burden on caregivers. One of the few assistive devices to aid in sit-to-stand is the Topro Taurus walker [33], which has the ability to lift patients up from sitting to standing position. It has arm rests mounted on an actuated telescopic shaft that rises vertically (Figure 1(c)), and thus differs significantly from standard walkers. When a person resting on the arm rests activates a lift switch, the walker height increases and it raises their upper body.

Body part	Mass (kg)	Moment-of-Inertia in sagittal plane (kg-m^2)
Lower leg	8.96	0.828
Upper leg	16.9	0.83
Head and trunk	46.28	2.564
Upper arm	4.46	0.268
Lower arm	3.82	0.317

TABLE I: Physical parameters of different body parts of the human model [17].

We propose a device that can follow the typical trajectory of a human rising up from sitting to standing posture while providing the necessary support (forces and torques) to minimize the effort. We model the human as a system of rigid links connected by revolute joints, along with an assistive device D as shown in Figure 1. The human is hinged at the ankle and the device D is hinged at its base. The device D has an actuator at its base (Figure 1) that can rotate it, an actuated telescopic shaft, and arm rests that are always kept horizontal by using a parallelogram mechanism. Thus the proposed assistive device (Figure 1 (b)) has one more actuated degree of freedom than the existing Topro walker.

When standing up, we minimize the human effort, approximated by the sum of squared torques of all the joints over the duration of the sit-to-stand action. This is the objective function in our optimization problem (Equation (12)). Figures 2 (a) and (b) show the human with device D in sitting and standing positions. The trajectory optimization problem of sit-to-stand with the assistive device involves two systems, the human, and the device D. The systems interact where the human holds the grab bars and on the arm rests of D (Figures 2 (c) and (d)). Figure 2(d) shows the free-body diagrams of the forearm and arm rest and the action-reaction wrenches $F_{x_{cl}}, F_{y_{cl}}, T_c$ on each system. $F_{x_{cl}}, F_{y_{cl}}$ are the Lagrange multipliers associated with the loop-closure constraint (where the human holds the grab bars of D) and the reaction torque T_c is the Lagrange multiplier associated with the constraint that the human forearm and the arm rest are always horizontal throughout sit-to-stand, i.e., $\theta_{forearm} = \pi$ rads.

For the human, the actuation torque at joint j therefore now has additional terms to be added to the expression for torque in Equation (4). We now have

$$\tau_j = \begin{cases} \tau_j^{(4)} + F_{y_{cl}} l_j c_j - F_{x_{cl}} l_j s_j & \text{for } j < n \\ \tau_j^{(4)} + F_{y_{cl}} l_j c_j - F_{x_{cl}} l_j s_j + T_c & \text{for } j = n \end{cases} \tag{13}$$

where $\tau_j^{(u)}$ denotes the expression for torque τ_j in Equation (u). We minimize $\sum_{i=1}^{n_H} \tau_i^2$, where n_H is the number of joints in the human model, by solving the nonlinear optimization problem (Equation (12)), and find the optimal control points of the cubic B-splines for the joint configurations $\boldsymbol{\theta}$ of the human and variables $F_{x_{cl}}, F_{y_{cl}}, T_c$. Since the human and D form a closed loop, the accelerations of the human hand and the grab bar of D (Figure 2 (c)) are the same. From the acceleration of the grab bar of D, we calculate the angular and linear accelerations of the revolute and prismatic joints respectively of D. We then use inverse dynamics to calculate the corresponding actuation wrenches for D.

$$F_D = m_{II}\ddot{x}_D + m_{II}g - F_{x_{cl}}c_D - F_{y_{cl}}s_D \tag{14}$$

$$\begin{aligned} T_D = &(I_I + m_I r_I^2)\ddot{\theta}_D + m_{II}(x_D + r_{II})^2\ddot{\theta}_D \\ &+ 2m_{II}(x_D + r_{II})\dot{x}_D\dot{\theta}_D \\ &+ \{m_I r_I + m_{II}(x_D + r_{II})\} g c_D \\ &+ (x_D + l_{II})(-F_{y_{cl}}s_D + F_{x_{cl}}c_D) + T_c \end{aligned} \tag{15}$$

where $\ddot{x}_D$ is the linear acceleration of link II and $\ddot{\theta}_D$ is the angular acceleration of link I of D calculated from the acceleration a_{cl} of the hand-tip of the human (point P in Figure 1(a)) that coincides with the grab-bar of the device (Figure 2(d)). θ_D is the angular displacement of I, T_D is the actuation torque at joint I of D, and F_D is the linear actuation force acting on II. $m_{\{I,II\}}$, $I_{\{I,II\}}$, $r_{\{I,II\}}$, and $l_{\{I,II\}}$ are the mass, moment of inertia about the COM, distance of COM from proximal joint, and length of links I and II of D respectively (Figure 1). Links I and II are modeled as hollow

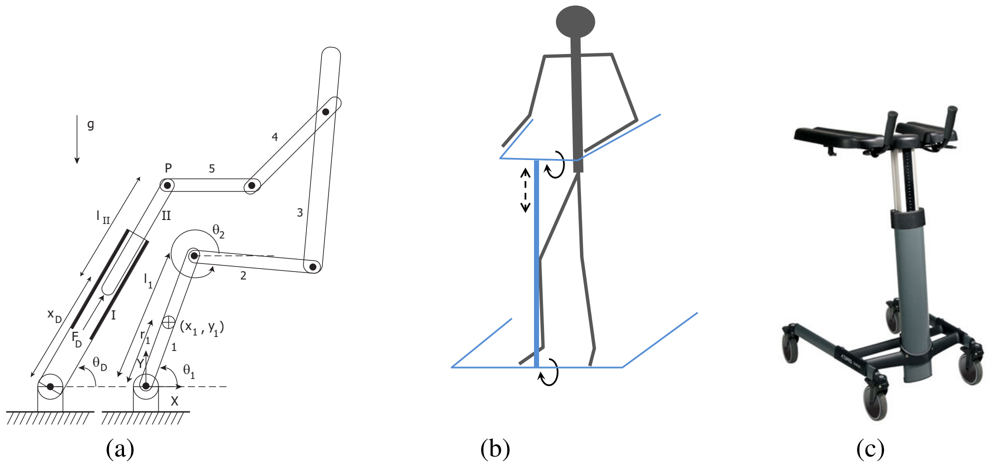

Fig. 1: (a) A rigid link model of a human with the proposed assistive device. The assistive device D consists of a telescopic shaft that can rotate about its base and horizontal arm rests. The body links of the human are numbered using Indo-Arabic numerals, and those of the assistive device are numbered using Roman numerals. Both the human and the device are hinged about their bases. (b) Schematic sketch of assistive device with rotational degrees of freedom for the forearm rest (unactuated) and the lift mechanism (actuated). (c) Topro Taurus walker [33] with a prismatic lift mechanism.

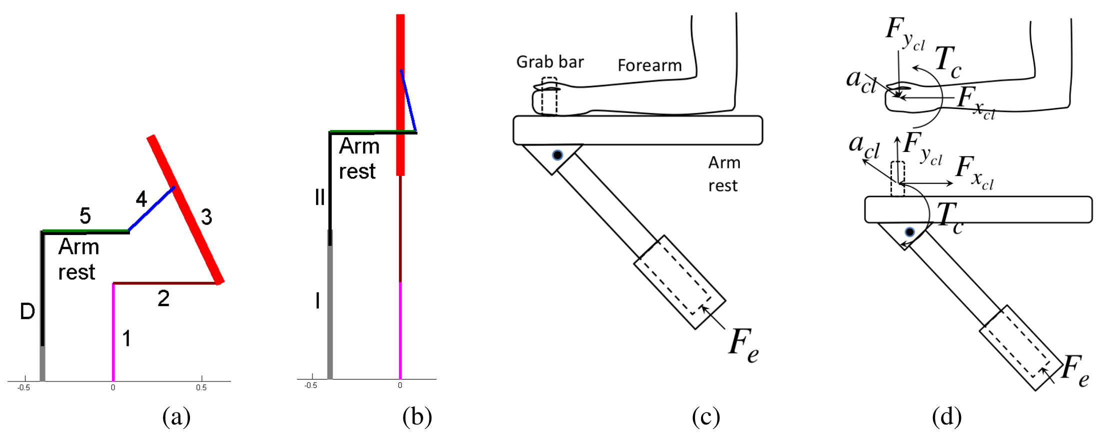

Fig. 2: (a) and (b) Human with assistive device D in the sagittal plane. Different parts of the human are numbered and colored. 1: Lower leg (magenta) 2: Upper leg (dark brown) 3: Head and torso (red) 4: Upper arm (blue) 5: Lower arm (green). (a) Sitting posture with D. (b) Standing posture with D. The female (grey, I) and male (black, II) parts of the prismatic joint in D are labeled. (c) The forearm of the human holding the assistive device's arm rest. (d) The free body diagrams of the forearm and the arm rest. The reaction wrenches $F_{x_{cl}}, F_{y_{cl}}, T_c$ are shown with directions.

and solid cylinders of mass 12 kg and 6 kg respectively, $l_I = 0.85$ m, and $l_{II} = 0.8$ m.

We calculated the optimal trajectories and joint actuation wrenches of the human and device D for a human with parameters in Table I weighing 80.42 kg (788.92 N). The optimal postures of a human standing up with the help of device D are shown in Figure 3. The total time for sit-to-stand is assumed to be 4 s [31]. The trajectory of each body part is modeled using uniform cubic B-splines with eight control points having specified end configurations and zero end velocities and accelerations. Eight is the minimum number of control points required to satisfy the boundary conditions.

Figure 3 shows the human standing up with two types of devices. The top row shows our proposed device D following a trajectory that minimizes the effort of the human for sit-to-stand; the value of the objective function (i.e., sum of squares of torques in the human) is 1.252×10^4 N^{2}m^2. The bottom row shows the human standing up using a device that has no rotational degree of freedom at its base; here the value of the objective function is 1.934×10^4 N^{2}m^2. Figure 4 shows the wrenches for our device and the device with only linear actuation. The peak values for both the actuation forces and torques are larger for the device with only linear actuation. (Note that for this device, the torque is an action-reaction torque and not applied by any actuator.) Further, the

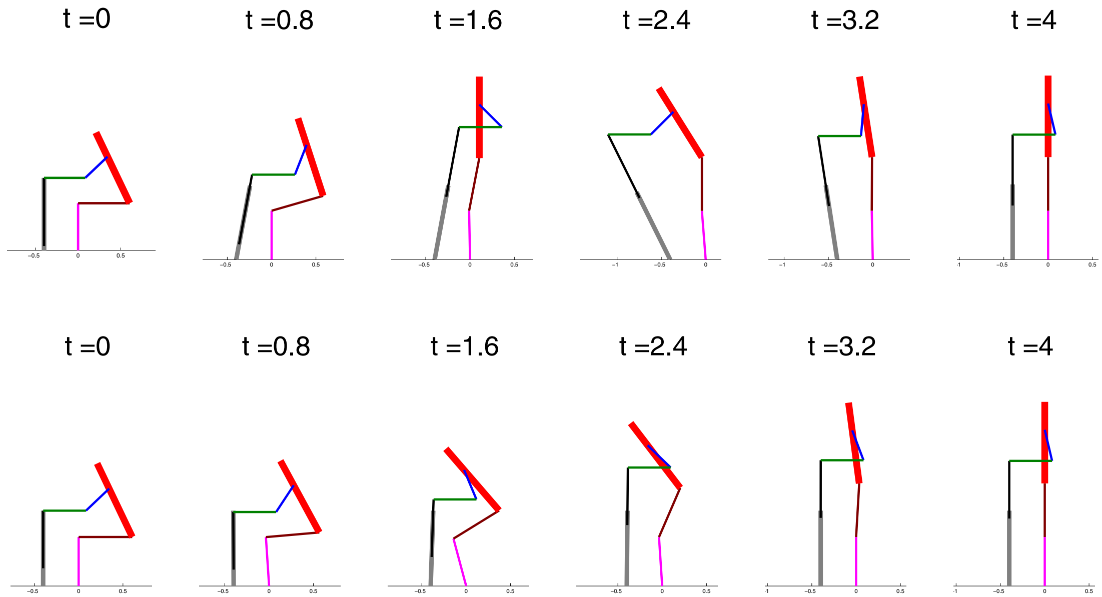

Fig. 3: Human standing up with help of assistive device, shown at 0.8 s intervals. Top row: Sit-to-stand with an assistive device that follows the human's trajectory. Bottom row: Sit-to-stand with a device that rises almost vertically.

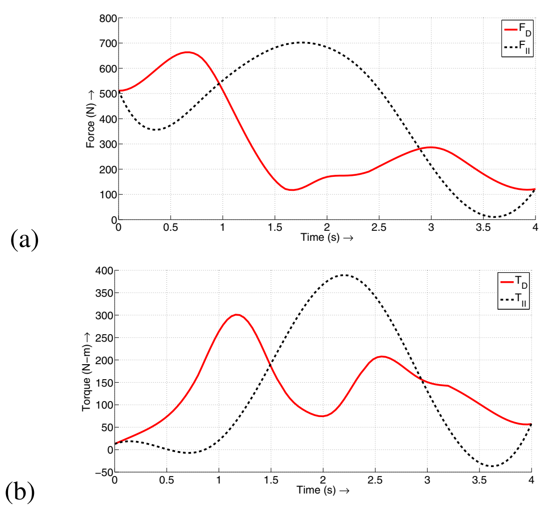

Fig. 4: Plots of sit-to-stand assistive device actuation wrenches for device D and for a device with only a linear actuator that lifts the human vertically. (a) Force in the linear actuator. (b) Torque at the base of the device.

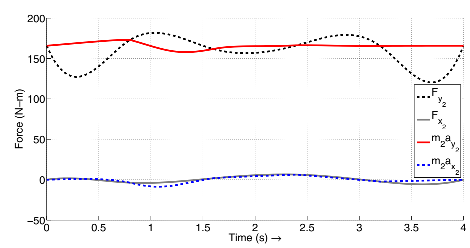

Fig. 5: Plot of force variables for joint 2 from Equations (5) and (6). Note that the constraints are guaranteed to be satisfied at the knots {0.0, 0.8, 1.6, 2.4, 3.2, 4.0}.

wrenches in device D are more evenly distributed over the entire sit-to-stand action; this may qualitatively explain why the effort required by the human is lower with device D. Hence we conclude that the proposed assistive device with two degrees of freedom (a rotational degree of freedom at the base in addition to a translational degree of freedom) can more effectively help an injured or elderly human during sit-to-stand than a device with only one translational degree of freedom like the Topro walker.

We also considered sit-to-stand of an injured human where we limit the maximum force on the knees to 200 N. For the injured human we use the constrained Lagrangian formulation (Equation (2)) updated with loop-closure and arm rest constraints. The actuation wrenches for device D are calculated using inverse dynamics as before (Equations (14) and (15)). We additionally have to apply the constraints of Equations (5) and (6). We adopt a collocation procedure in which these equations are incorporated as constraints in the nonlinear trajectory optimization problem (Equation (12)). Figure 5 plots the force variables of Equations (5) and (6). The constraints are guaranteed to be satisfied only at the knots, corresponding to 0, 0.8, 1.6, 2.4, 3.2, and 4.0 seconds.

Using our combined approach of trajectory optimization and dynamics, the actuation and constraint wrenches of the device as well as its configuration and acceleration at all instants of time during sit-to-stand can be determined.

V. Conclusion

We have presented a trajectory optimization formulation that uses a constrained Lagrangian to incorporate constraints on internal joint reaction wrenches. We apply the formulation to generate human sit-to-stand motions, and use them to derive kinematic and dynamic parameters for the design of an assistive device to minimize the effort required of the human during sit-to-stand. We compared our proposed device with the model of a commercial device and showed that the added actuated degree of rotational freedom can reduce the human effort. Further, our trajectory optimization formulation can incorporate bounds on internal reaction wrenches at specified joints to model motion impairments arising from infirmity or injury. More broadly, the formulation can be used as a design tool for the integrated design and trajectory optimization of robotic systems.

Our future work will focus on the fabrication and experimental evaluation of the sit-to-stand assistive device and investigate satisfaction of the dynamics constraints along the trajectory.

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